PHYSICS 2DL – SPRING 2010

MODERN PHYSICS LABORATORY

Monday May 24, 2010

Course Week 9

Last Lecture

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UCSD

UCSD

2DL Review

Final Exam
Closed Book, But You Can Bring:

1. Calculator (Scientific)

2. one 8.5" x 11" sheet of notes, HANDWRITTEN, front and back

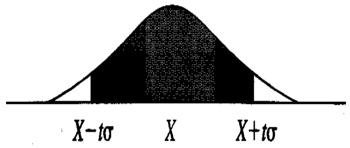
Topics

- Averages
- Variance
- Distributions
- Gaussian
- Least squares
- t-test
- Chi-Sq
- Binomial
- Poisson

+ Other Topics & Statistics of the Six 2DL expts

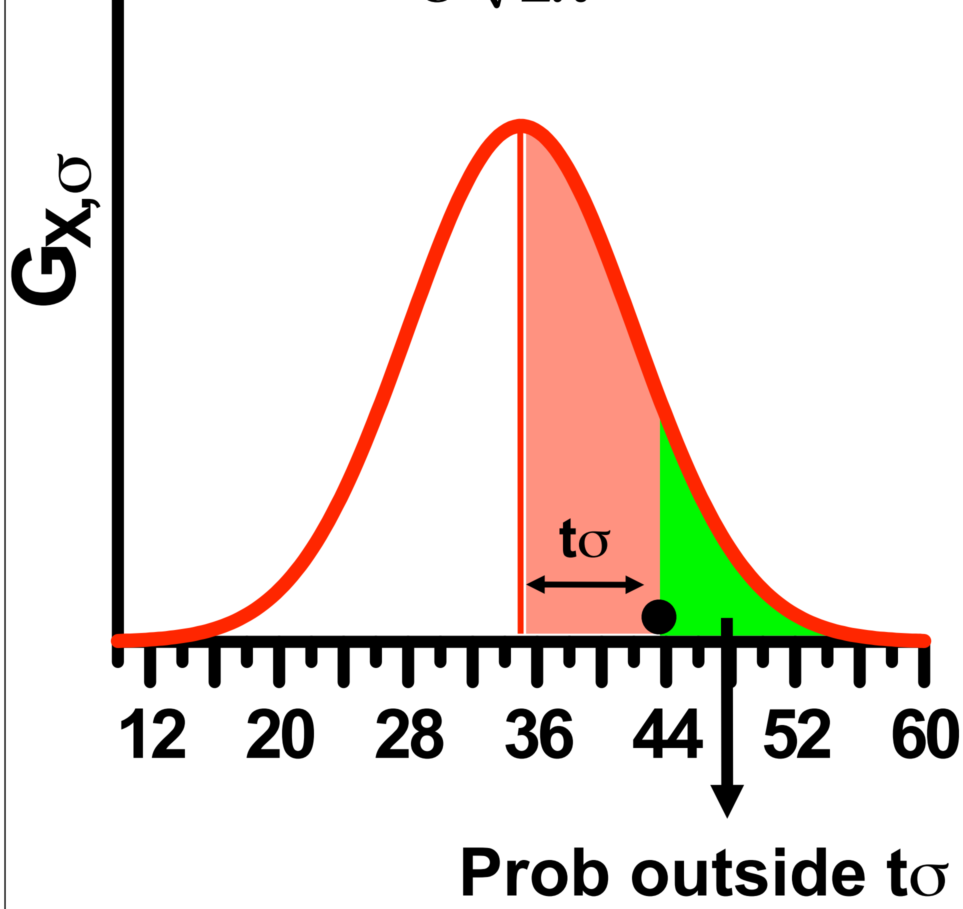
χ^2 TEST for FIT

Table A. The percentage probability, $Prob(\text{within } t\sigma) = \int_{X-t\sigma}^{X+t\sigma} G_{X,\sigma}(x)\,dx$, as a function of t.



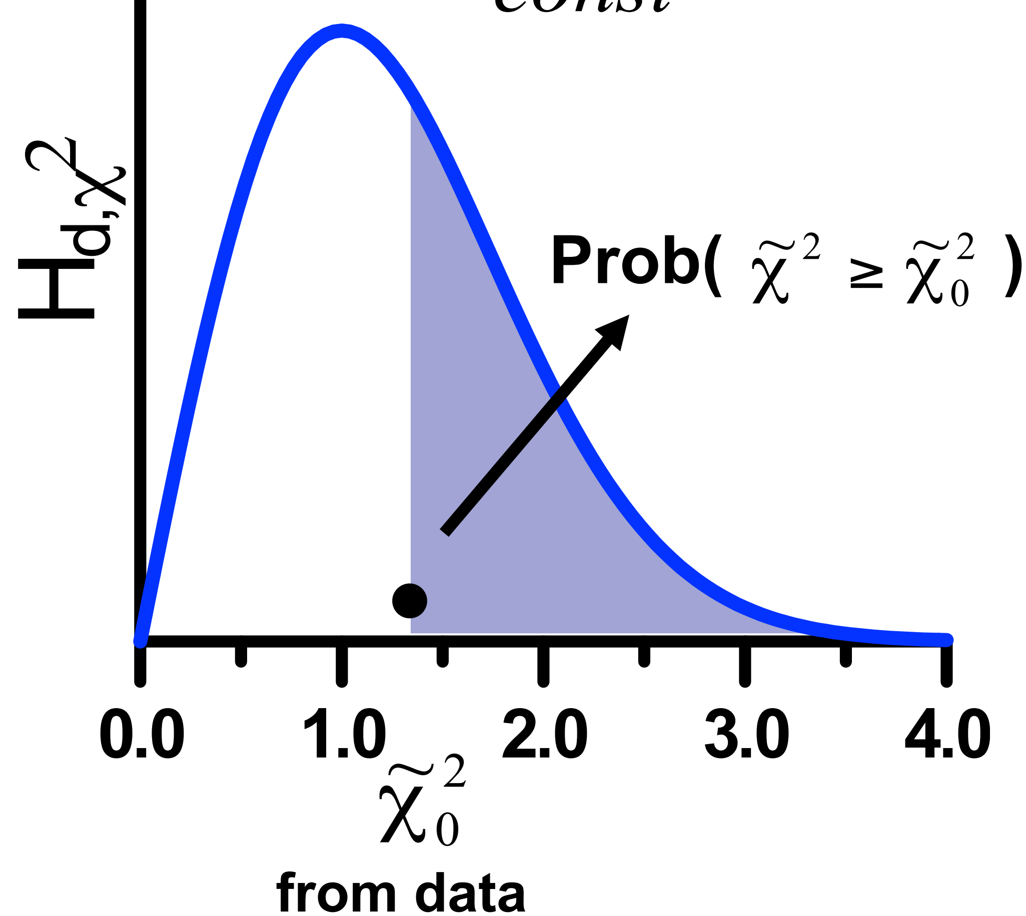
Gauss distribution:

$$G_{X,\sigma} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-X)^2/2\sigma^2}$$



$\tilde{\chi}^2$ distribution:

$$H_{d,\tilde{\chi}^2} = \frac{1}{const} x^{d-1} e^{-x^2/2}$$



A student measures the force on a length of wire (**g**) on a current carrying wire in a magnetic field versus the current **I** through the wire

F/g (Force/Unit Length)

The tabulated data and uncertainties are shown below.

I, (A)	F/g (N/m)
0	0±1
1	12±1
2	21±1
3	30±1
4	39±1
5	50±1

a) The student wants to perform a least-square fit to the theory relation $F/g = B \times I$. Write an expression for χ^2 and DERIVE an expression for B in terms of the data values.

b) What is the best-fit value of B for the data?

c) What is the reduced value of $\tilde{\chi}_0^2$?

d) Using the table from Appendix D assess the agreement between the fit and the theory.

a.

$$\chi^2 = \frac{\sum_{j=1}^{N}\left(y_j - f(x_j)\right)^2}{\sigma_y^2} \qquad \frac{F}{g} = B \times I \qquad \chi^2 = \frac{\sum_{j=1}^{N}\left(\frac{F_j}{g} - B \times I_j\right)^2}{\sigma_y^2}$$

b.

$$\frac{\partial \chi^2}{\partial B} = 0 \qquad \frac{\partial \chi^2}{\partial B} = \frac{1}{\sigma_y^2}\sum_{j=1}^{N} 2\left[\left(\frac{F_j}{g} - B \times I_j\right)\left(-I_j\right)\right] = 0$$

$$\sum_{j=1}^{N}\left[\frac{F_j}{g} \times -I_j\right] + \sum_{j=1}^{N}\left[\left(-BI_j\right)\left(-I_j\right)\right] = 0$$

$$B = \frac{\sum_{j=1}^{N} I_j \frac{F_j}{g}}{\sum_{j=1}^{N} I_j^2} \qquad B = 10$$

c.

$$\tilde{\chi}_0^2 = \frac{\chi^2}{d} = \frac{\chi^2}{6-1} = 1.2$$

Table D

d.

$$P\left(\tilde{\chi}^2 > \tilde{\chi}_0^2\right) = 31\%$$

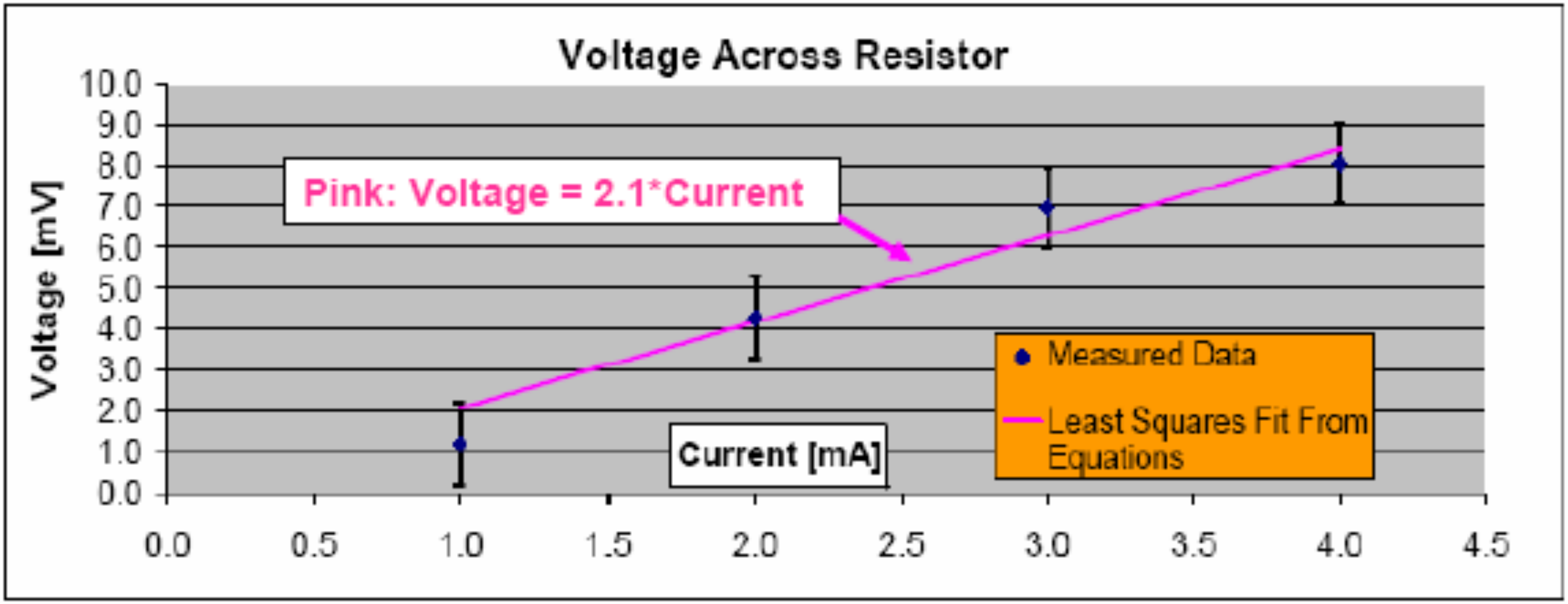
Table D. The percentage probability $Prob_d(\tilde{\chi}^2 \geqslant \tilde{\chi}_o^2)$ of obtaining a value of $\tilde{\chi}^2 \geqslant \tilde{\chi}_o^2$ in an experiment with d degrees of freedom, as a function of d and $\tilde{\chi}_o^2$. (Blanks indicate probabilities less than 0.05%.)

$\tilde{\chi}_o^2$

d	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	8.0	10.0
1	100	48	32	22	16	11	8.3	6.1	4.6	3.4	2.5	1.9	1.4	0.5	0.2
2	100	61	37	22	14	8.2	5.0	3.0	1.8	1.1	0.7	0.4	0.2		
3	100	68	39	21	11	5.8	2.9	1.5	0.7	0.4	0.2	0.1			
4	100	74	41	20	9.2	4.0	1.7	0.7	0.3	0.1	0.1				
5	100	78	42	19	7.5	2.9	1.0	0.4	0.1						

d	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
1	100	65	53	44	37	32	27	24	21	18	16	14	12	11	9.4	8.3
2	100	82	67	55	45	37	30	25	20	17	14	11	9.1	7.4	6.1	5.0
3	100	90	75	61	49	39	31	24	19	14	11	8.6	6.6	5.0	3.8	2.9
4	100	94	81	66	52	41	[illegible]	23	17	13	9.2	6.6	4.8	3.4	2.4	1.7
5	100	96	85	70	55	42	31	22	16	11	7.5	5.1	3.5	2.3	1.6	1.0
6	100	98	88	73	57	42	30	21	14	9.5	6.2	4.0	2.5	1.6	1.0	0.6
7	100	99	90	76	59	43	30	20	13	8.2	5.1	3.1	1.9	1.1	0.7	0.4
8	100	99	92	78	60	43	29	19	12	7.2	4.2	2.4	1.4	0.8	0.4	0.2
9	100	99	94	80	62	44	29	18	11	6.3	3.5	1.9	1.0	0.5	0.3	0.1
10	100	100	95	82	63	44	29	17	10	5.5	2.9	1.5	0.8	0.4	0.2	0.1
11	100	100	96	83	64	44	28	16	9.1	4.8	2.4	1.2	0.6	0.3	0.1	0.1
12	100	100	96	84	65	45	28	16	8.4	4.2	2.0	0.9	0.4	0.2	0.1	
13	100	100	97	86	66	45	27	15	7.7	3.7	1.7	0.7	0.3	0.1	0.1	
14	100	100	98	87	67	45	27	14	7.1	3.3	1.4	0.6	0.2	0.1		
15	100	100	98	88	68	45	26	14	6.5	2.9	1.2	0.5	0.2	0.1		
16	100	100	98	89	69	45	26	13	6.0	2.5	1.0	0.4	0.1			
17	100	100	99	90	70	45	25	12	5.5	2.2	0.8	0.3	0.1			
18	100	100	99	90	70	46	25	12	5.1	2.0	0.7	0.2	0.1			
19	100	100	99	91	71	46	25	11	4.7	1.7	0.6	0.2	0.1			
20	100	100	99	92	72	46	24	11	4.3	1.5	0.5	0.1				

Here's our "final" example of the general technique when fitting for only a slope….



Fitting Voltage Data to V=IR

$$\frac{\partial \chi^2}{\partial R} = 0$$

$$IMPLIES:$$

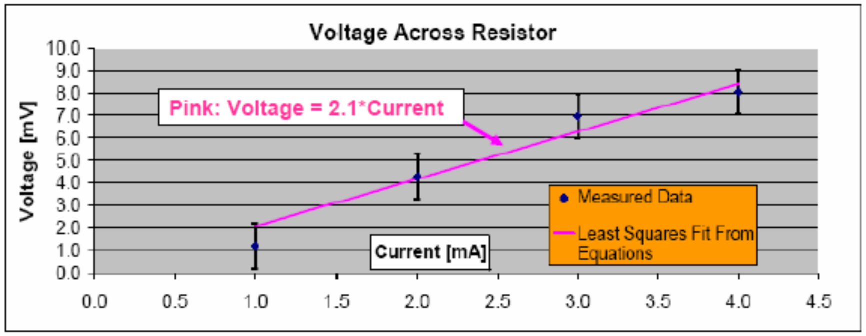
N = number of data points. In this example, N=4

$$R = \frac{\sum_{i}^{N} I_i V_i}{\sum_{i}^{N} I^2{}_i}$$

See Taylor

Problem 8.18

Don't Use Linear fit with A=0!



What is the Error on the Best-Fit Parameter R?

Our general formula, which always applies, is:

$$\sigma_R = \sqrt{\left(\frac{\partial R}{\partial V_1}\right)^2 \sigma_{v_1}^2 + \left(\frac{\partial R}{\partial V_2}\right)^2 \sigma_{v_2}^2 + \ldots \left(\frac{\partial R}{\partial V_N}\right)^2 \sigma_{v_N}^2}$$

Since:

$$\left(\frac{\partial R}{\partial V_1}\right)^2 = I_1^2, \left(\frac{\partial R}{\partial V_N}\right)^2 = I_N^2$$

$$and : \sigma_{V_N} = 1mV$$

Putting it all together:

$$so : \sigma_R = \frac{1mV \sqrt{\sum_i^N I_i^2}}{\sum_i^N I_i^2}$$

Check units are right, error has same units as R.

LEAST SQUARES FITTING EXAMPLE

current [mA]	voltage [mV]	voltage error [mV]	voltage measured [mV]	voltage uncertainty [mV]	x^2 [mA^2]	xy [mA*mV]	voltage from fit [mV]
1.0	2.0	-0.8	1.2	1.0	1.0	1.2	2.1
2.0	4.0	0.3	4.3	1.0	4.0	8.6	4.2
3.0	6.0	1.0	7.0	1.0	9.0	20.9	6.3
4.0	8.0	0.0	8.0	1.0	16.0	32.1	8.4
this is "x_i"			this is "y_i"	this is "σ_y"	$\sum x^2$ 30.0	$\sum xy$ 62.9	

This is the true signal

This is the true signal with error (uncertainty).

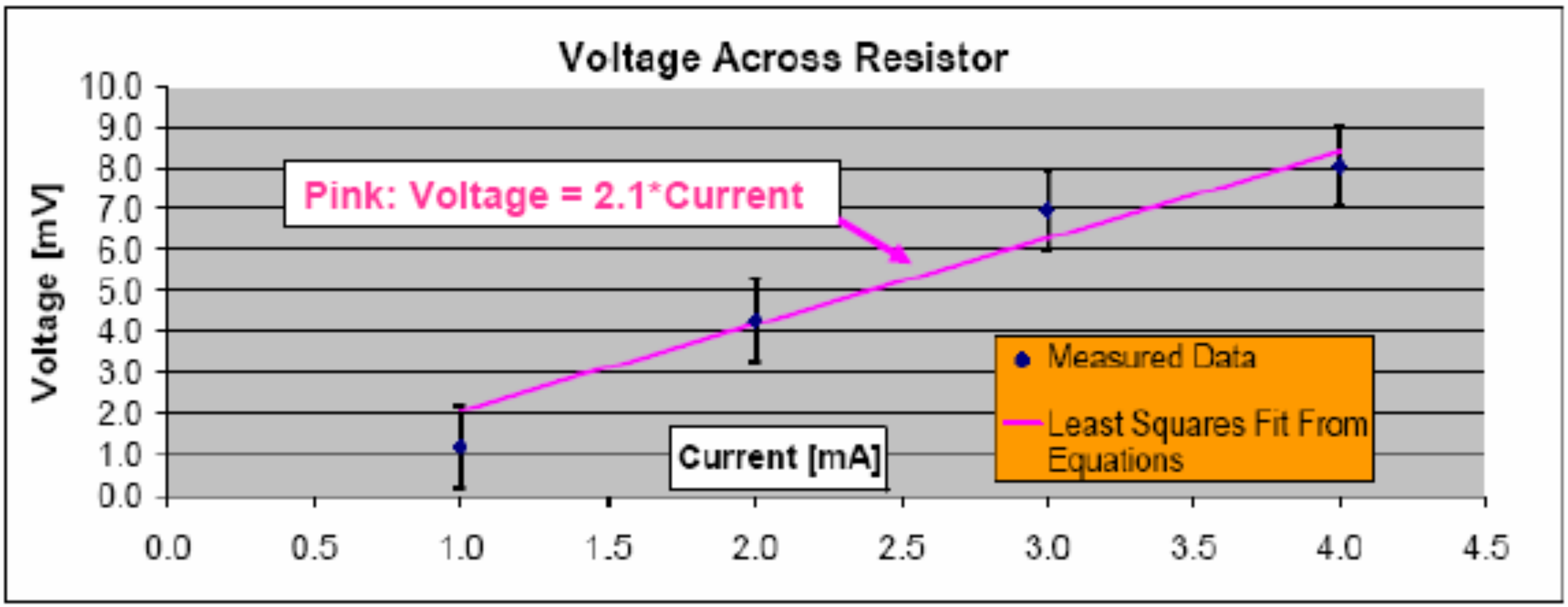
What we would measure in real-life

Our model: V=I×R

R from Fit: $R = \sum(xy)/\sum(x^2)$ 2.1 Ω

Error in R comes from partial derivative of numerator with respect to y_i only

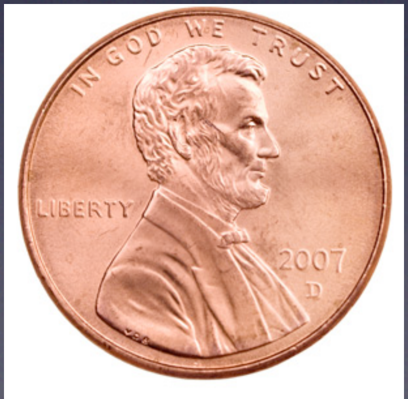
Error in R $\sigma_R = \sigma_y / \sqrt{(\sum x^2)}$ 0.2 Ω



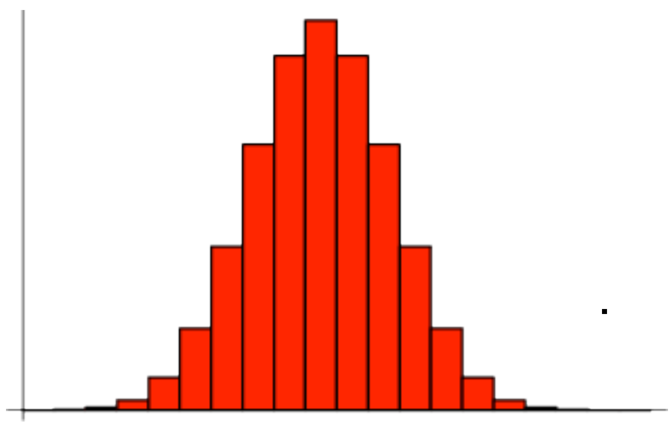
Ch 10 Binomial Distribution

Why Binomial? Because only 2 outcomes of a given test. Either X happened or it didn't, where 'X' can be a complicated statement like:

"When throwing 3 coins sequentially, what's the probability that the sequence observed was HHT"



Ch 10 Binomial Distribution



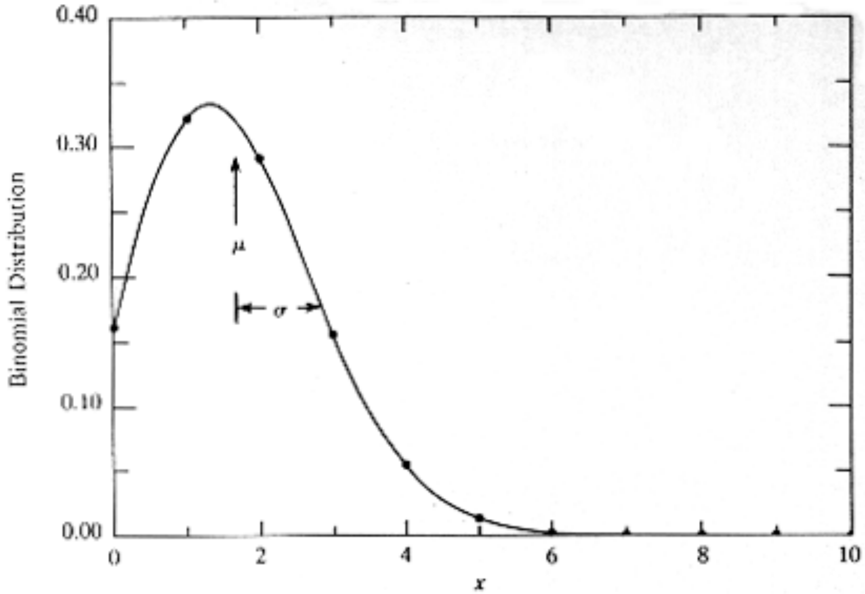
$$P_p(n|N) = \binom{N}{n} p^n q^{N-n}$$

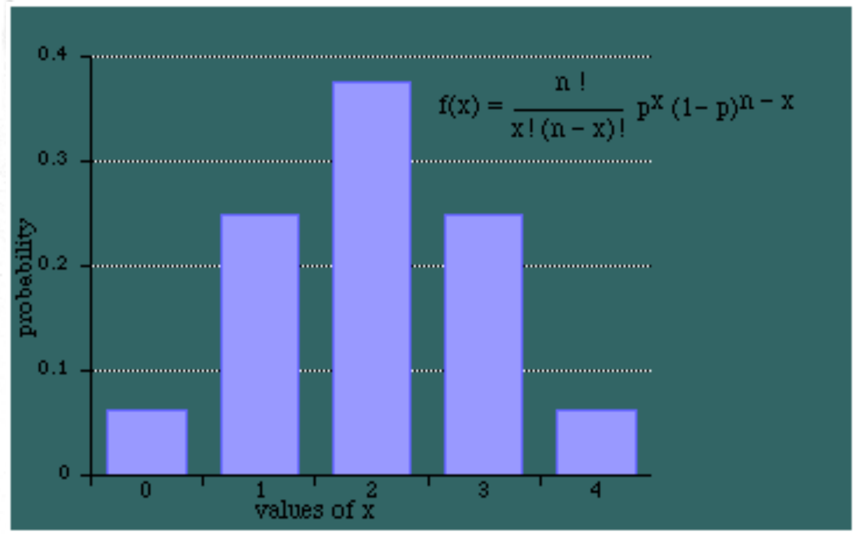
Binomial coefficient

20 trials, with $p = q = 1/2$

Symmetric only if p = q.

Ch 10 Binomial Distribution





The ***binomial distribution*** *describes the behavior of a count variable X if the following conditions apply:*

1: *The number of observations n is fixed.* ***2:*** *Each observation is independent.*

3: *Each observation represents one of two outcomes ("success" or "failure").*

4: *The probability of "success" p is the same for each outcome.*

Binomial Distributions in Practice

- You should really know when to use the Gaussian hypothesis. When the number of attempts/trials is > 15, you are safe.

Then :

$$\mu_X = np$$
$$\sigma_X^2 = np(1-p)$$

•Then use the one or two sided t-probability distributions to get the probability.

•This is nice also because calculating the factorial is very computationally demanding when N > 50.

Binomial Example

- What's the probability of getting 27 Heads out of 34 tosses of a coin?

$$B_{27,1/2}(\nu) = \frac{34!}{27!7}\left(\frac{1}{2}\right)^{27}$$

MICROSOFT EXCEL:

=BINOMDIST(23,36,0.5,FALSE)

$$G_{\overline{x}=17,\sigma=\sqrt{17(0.5)}}(\nu) = \frac{1}{2.6\sqrt{2\pi}}\exp\left[-\left(27-\overline{17}\right)^2 \Big/ 2(2.9)^2\right]$$

MICROSOFT EXCEL:

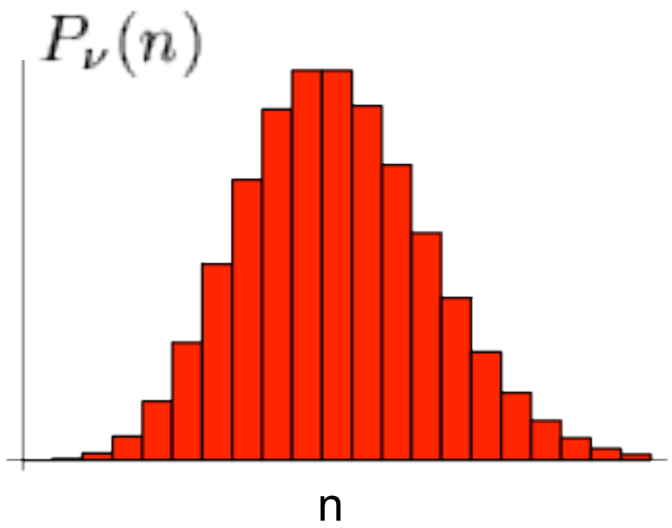
=NORMDIST(x,mean,standdev,FALSE)

http://www.stat.sc.edu/~west/javahtml/CLT.html

Ch 11 Poisson Distribution

Given a Poisson process, the probability of obtaining exactly n successes in N trials is given by the limit of a binomial distribution

$$P_p(n|N) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n}.$$



Now, instead of looking at getting n out of N if we define the number of of successes, we can set:

$$\nu \equiv Np$$

$$P_{\nu/N}(n\,|\,N) = \frac{N!}{n!\,(N-n)!}\left(\frac{\nu}{N}\right)^n\left(1-\frac{\nu}{N}\right)^{N-n}.$$

$$P_\nu(n) = \lim_{N\to\infty} P_p(n \mid N)$$

$$= \lim_{N\to\infty} \frac{N(N-1)\cdots(N-n+1)}{n!} \frac{\nu^n}{N^n} \left(1 - \frac{\nu}{N}\right)^N \left(1 - \frac{\nu}{N}\right)^{-n}$$

$$= \lim_{N\to\infty} \frac{N(N-1)\cdots(N-n+1)}{N^n} \frac{\nu^n}{n!} \left(1 - \frac{\nu}{N}\right)^N \left(1 - \frac{\nu}{N}\right)^{-n}$$

$$= 1 \cdot \frac{\nu^n}{n!} \cdot e^{-\nu} \cdot 1$$

$$P_\nu(n) = \frac{\nu^n e^{-\nu}}{n!},$$

Poisson – again should know when to use Gaussian

- Because Poisson is hard to calculate (with factorial) and because it's easier to find probability tabulated for Gaussian distribution.
- Approximation to use:

$$\overline{X} = \mu$$

$$\sigma = \sqrt{\mu}$$

- Then use t-values to answer likelihood questions.
- Caveat- N must be large! Better if large and symmetric.

e/m for the electron

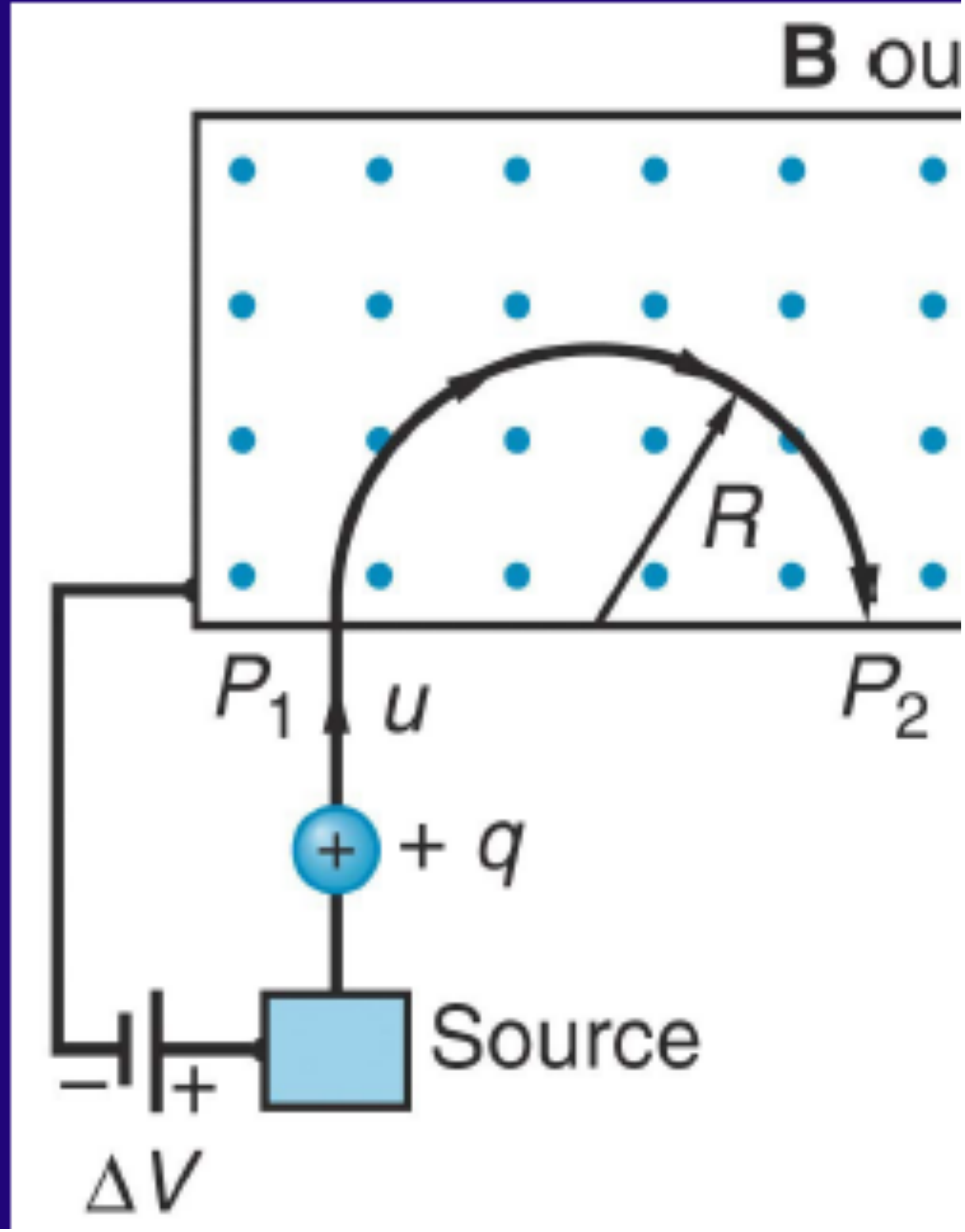
Determination of e/m for Electron

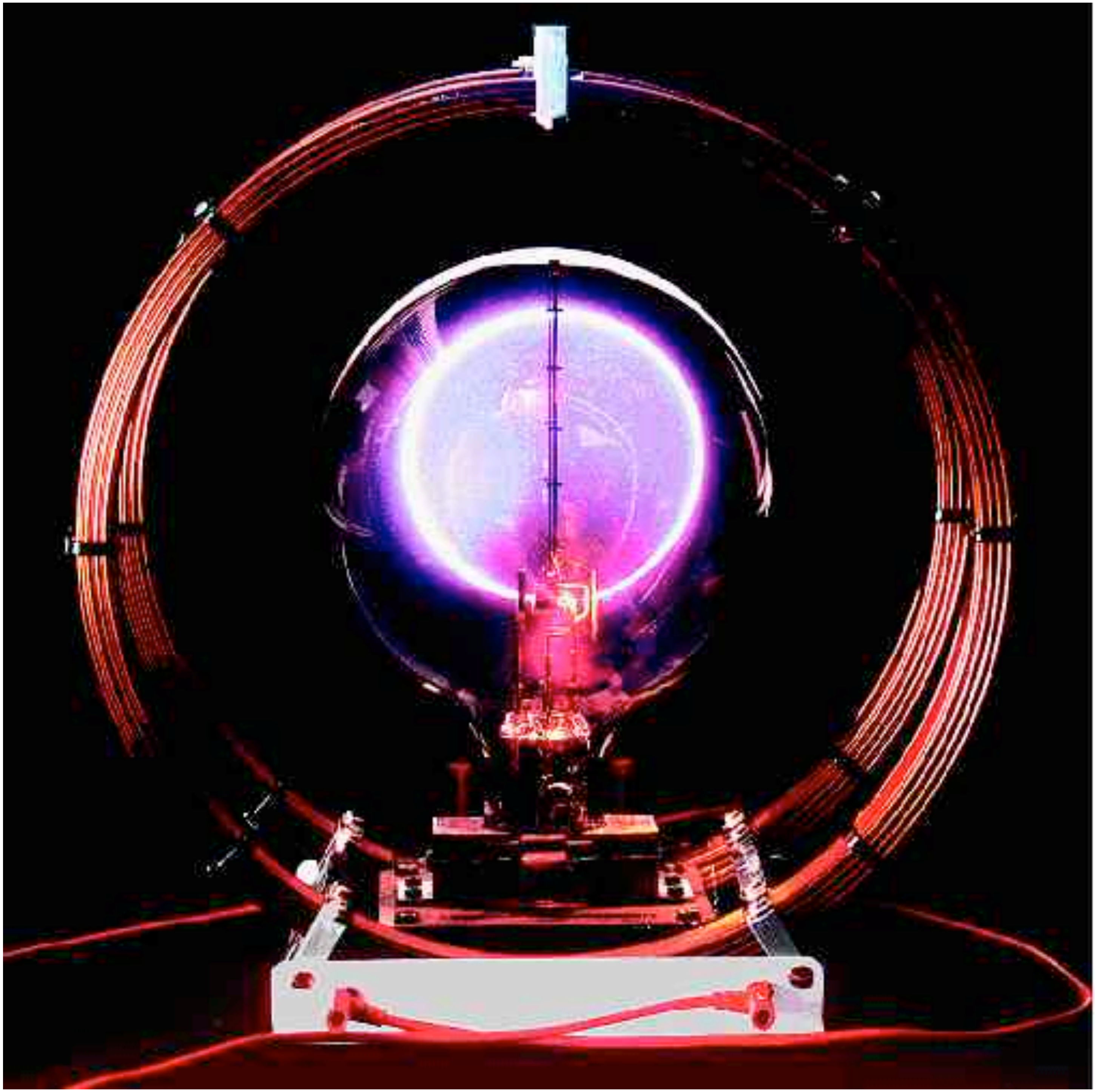
- e/m is characteristic of a particle : electron Vs Cl^- ion
- When Uniform magnetic field of strength B is established perpendicular to direction of motion of a charged particle, particle moves in a circular path of radius R

$$quB = \frac{mu^2}{R} \Rightarrow R = \frac{mu}{qB}$$

If electrons have KE = qV

then $$\frac{e}{m} = \frac{2V}{B^2R^2}$$



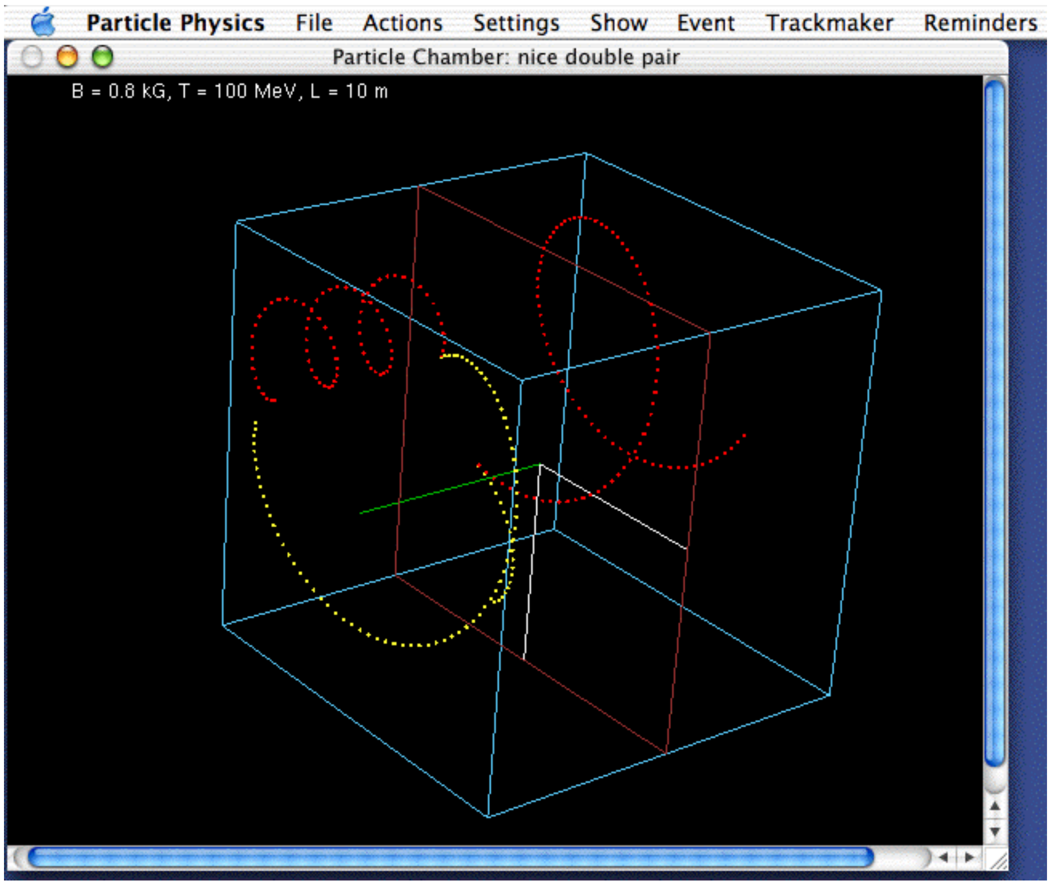


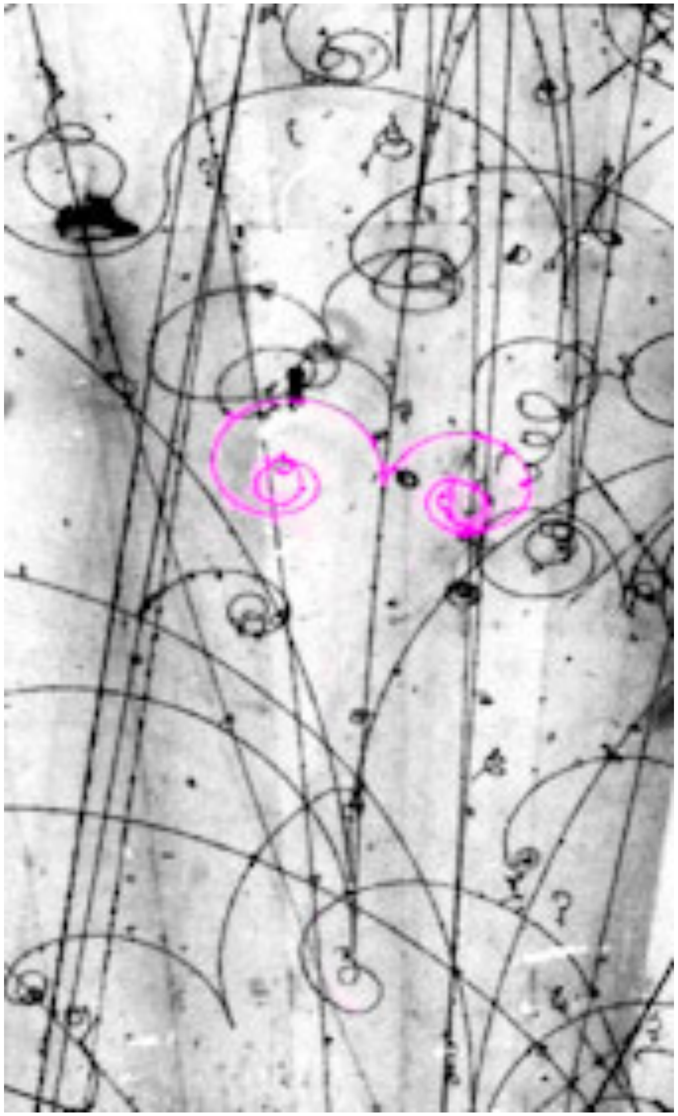
e/m, e, m of Electron : Why Important

Realization that electron is much less massive than the Hydrogen atom made physicists think about the structure Inside atom

The electron was discovered just a bit over 100 years ago, triggered A scientific revolution

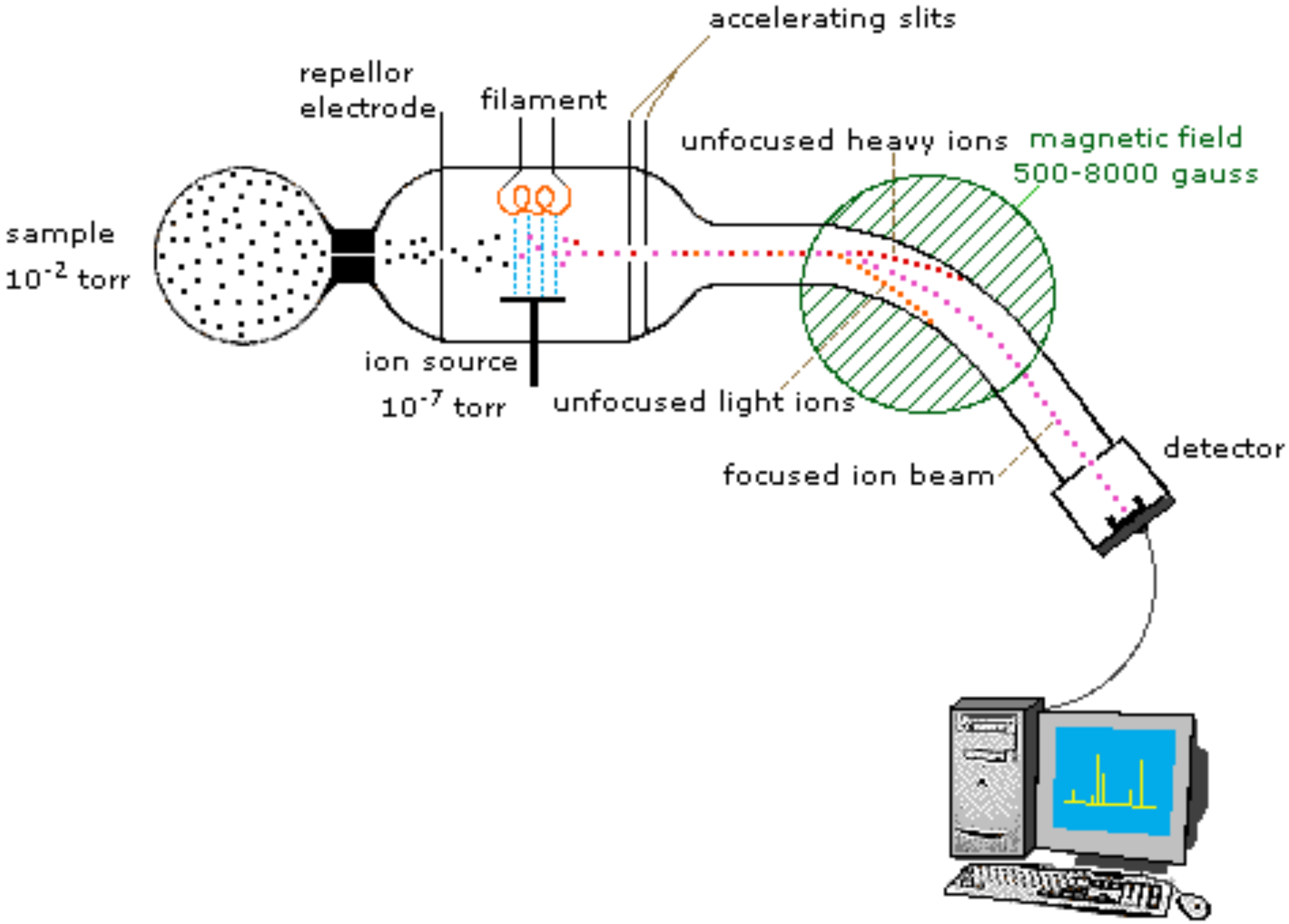
Thomson's idea
Still used to measure
Masses of fundamen
Particles or nuclei





Electron-Positron Pair

Mass Spectrometer



GOOD LUCK

- Email me or TA’s with any questions

- HAVE A GREAT SUMMER